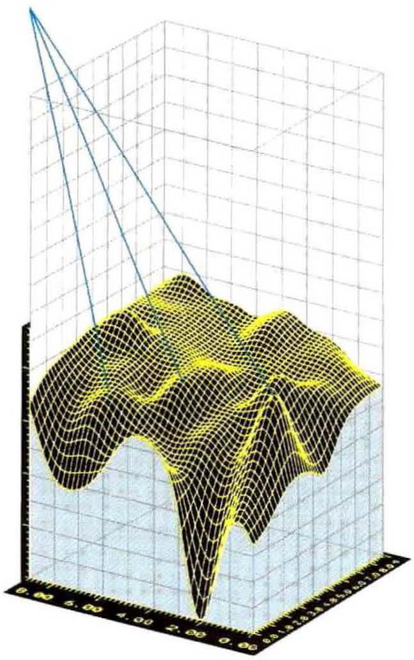

(Note correction to absolute zero temperature.)

GPS Receiver System Noise

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How well a GPS receiver performs — that is, how precisely it can measure the pseudorange and carrier phase — largely depends on how much noise accompanies the signals in the receiver's tracking loops. The more noise, the worse the performance. This noise either comes from the receiver electronics itself or is picked up by the receiver's antenna. In this month's article, we'll take a look at noise, discuss its causes, and assess its effect on the GPS observables.

"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as about the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who appreciates receiving your comments as well as suggestions of topics for future columns. To contact him, see the "Columnists" section on page 4 of this issue.

GPS receivers are not perfect devices — they cannot measure the GPS observables with infinite precision. They are always subjected to some level of noise that contaminates the observations.

What do we mean by *noise?* The common usage of the word is to describe a loud, harsh, or undesired sound. But the concept of noise extends to electrical signals too, whether or not those signals are meant for our ears. A more applicable definition would be any unwanted disturbances, superimposed on a signal, that tend to obscure a signal's usefulness or information content. So, in addition to the static or hissing that can emanate from stereo speakers, we have, for example, the "snow" on a television screen and "bit hits" in data transmissions.

In radio-based communication and navigation systems, noise originates from various sources: the equipment itself; other electrical devices such as radio transmitters, microwave ovens, motors, relays, ignition systems, and light dimmers; and natural terrestrial and extraterrestrial sources. We often call noise, particularly if it is from a human-made device, radio-frequency (RF) or electromagnetic interference.

This month, we will consider only noise generated by natural causes and by the receiver itself and save the subject of interference for a future article. Natural noise sources usually determine a receiver's minimum detectable or usable signal levels when it is operating in an environment free of human-made noise sources. These sources play a major role in determining receiver performance. In this article, we will first examine the origins of noise and how we quantify it, then assess the effect of noise on pseudorange and carrier-phase measurements.

THERMAL NOISE

The most basic kind of electrical noise is produced by the random movement of electrons in any conductor (including electronic components such as the resistors and semiconductors in a GPS receiver) with a temperature above absolute zero (0 K or −273.16 °C). We designate the resulting electromotive force or voltage *thermal noise* (or thermal agitation noise, resistor noise, or Johnson noise after J.B. Johnson, who analyzed it in 1928).

If we measure the voltage, we would see that it fluctuates rapidly, alternating in sign, such that the mean or average is zero. Yet the power associated with it — which is proportional to the integral of the square of the voltage — although small, is nonzero.

This available noise power, it turns out, is proportional to the conductor's absolute temperature. The noise occupies a broad frequency spectrum, and its power in a given passband is independent of the passband's center frequency. We refer to such noise as *white noise* in analogy to white light, which consists of more-or-less equal contributions of different colors. We can express the relationship between power, temperature, and bandwidth as

$$p = kTB \qquad [1]$$

where p is the thermal noise power, k is Boltzmann's constant ($1.380\,662 \times 10^{-23}$ joules [a joule is equal to a watt-second] per kelvin), T is the temperature in kelvins, and B is the bandwidth in hertz.

How much power are we talking about? Consider a resistor at room temperature (T = 290 K). The available power from the noise voltage in a 1-MHz bandwidth is $1.380\,662 \times 10^{-23} \times 290 \times 10^{6} \approx 4 \times 10^{-15}$ watts. Small indeed. But in considering the possible noise effect on signal use, we must compare the noise power with the available signal power. The power a received radio signal generates at an antenna's terminals can be quite small. For example, the GPS C/A-code signal generates only about 10^{-16} watts!

ANTENNA NOISE

The electrons randomly moving in a conductor produce not only noise voltage, but also electromagnetic radiation. In fact, all objects at temperatures above absolute zero radiate electromagnetic waves and may also absorb or reflect incident radiation. A perfect (hypothetical) absorber is called a *black body* — it absorbs all the incident radiation at all frequencies. The radiation energy heats the body to a particular temperature that is dependent on the radiation frequency.

A black body is also a perfect radiator. It emits a continuous radiation spectrum, with a brightness in the radio wavelength region given by

$$b = 2f^2kT/c^2 \qquad [2]$$

where f is frequency, k is Boltzmann's constant, T is the body's temperature, and c is the vacuum speed of light. If we could build a black body in the form of a box and place an antenna in it, the antenna would absorb the radiation emitted by the box walls and generate a noise power-per-unit bandwidth equal to kT — the same amount generated at the terminals of a resistor at temperature T. So, we can use the black body concept to characterize any electromagnetic radiation intercepted by an antenna in terms of the temperature of the black body that would have produced the same noise power.

Electromagnetic Radiation. A GPS receiver's antenna, like that of any radio receiver, picks up a certain amount of noise in the form of naturally produced electromagnetic radiation. This radiation comes from the sky, the ground, and objects in the antenna's vicinity.

The sky noise has two main components. The first, cosmic noise, is caused by the random electromagnetic radiation emitted by the sun, the Milky Way galaxy, and other discrete cosmic objects. Radiation left over from the Big Bang that gave birth to the universe some 10 billion years or so ago also contributes a little to this noise. The second source of sky noise is caused by the earth's atmosphere. The radiation from these sources is not confined to a narrow range of frequencies but extends over large portions of the radio spectrum. We can express the power of the radiation received from these sources in watts, but we find it more convenient to treat the radiation as if it were of thermal origin (whether or not that's actually the case) and express the power in terms of an equivalent *brightness temperature.*

Near the GPS frequencies, the galactic radiation noise temperature is quite small — about 10 K. The residual Big Bang radiation noise temperature is even smaller — 2.7 K. The sun, however, is quite bright at radio frequencies; at the GPS frequencies, it has an equivalent black body temperature of about 80,000 K during low solar activity (the quiet or undisturbed sun). During high solar activity (the disturbed sun), this value can be as great as perhaps 10,000,000 K. Interestingly enough, although the temperatures are quite high, solar radiation affects GPS performance negligibly, as we will see shortly.

The atmosphere is a source of noise because of its radiation absorption. In the 1–10-GHz frequency range, the noise temperature from atmospheric absorption varies from about 2 K at the zenith to about 90 K at the horizon. The actual values depend on factors such as the atmosphere's temperature profile and water-vapor concentration. Lightning from thunderstorms also produces electromagnetic radiation (sometimes called atmospherics), but the noise level drops rapidly with increasing frequency and is insignificant at the GPS frequencies.

The ground and objects in the vicinity of the antenna also radiate energy. The noise temperature of this radiation is approximately equal to the ground or object's actual temperature, with a typical value of 290 K.

ANTENNA TEMPERATURE

The sky and ground noise is intercepted by a GPS receiver's antenna along with the GPS signals. The amount of noise power from a particular source actually intercepted by an antenna — be it the sky, ground, or another object — depends on the direction from which the electromagnetic waves arrive and the gain of the antenna in that direction.

If the source of temperature T does not extend over the entire antenna reception pattern, the detected noise power, and hence temperature, will be less. This detected temperature is called the *antenna temperature.* If the antenna is a hypothetical isotropic type, with unit gain in all directions, then the antenna temperature is given by

$$T_a = \frac{\Omega_s}{4\pi} T_{avg} \qquad [3]$$

where Ω_s is the solid angular extent of the noise source, and T_{avg} is the average noise temperature of the source across the antenna reception pattern.

For the broad background sky noise and for the ground noise, Ω_s is 2π steradians, making the corresponding antenna temperatures about one-half the average temperature over the source. The sun subtends an angle of about 0.5 degrees so the corresponding isotropic antenna temperature at GPS frequencies is less than 0.5 K when the sun is quiet and only 6 K at most when the sun is disturbed.

GPS Antennas. Real GPS antennas are typically omnidirectional. Such an antenna has an essentially nondirectional pattern in azimuth and a directional pattern in elevation angle. At the zenith, the antenna typically has a few decibels of gain with respect to an isotropic antenna. The gain gradually drops down to a few decibels below that of an isotropic antenna at an elevation angle of 5 degrees or so. The resulting sky noise contributions to the antenna temperature therefore will be a little different from those calculated for an isotropic antenna but will be in the same ballpark.

A GPS antenna has sidelobes and backlobes that "look" at the warm earth. However, the gain of these lobes is typically quite small, so a zenith-pointing antenna sees only a small fraction of the ground noise temperature (but incline the antenna away from the zenith, and it will see more ground noise).

The various contributions to antenna temperature add algebraically, and a typical GPS antenna temperature might be around 130 K. This antenna temperature is the equivalent noise temperature of the antenna. If the antenna is replaced with a resistance equal to the antenna's impedance and cooled or heated up until it produces as much thermal noise as the antenna does when connected, then the resistance's temperature is the antenna's noise temperature. So the antenna noise temperature, T_a, is a measure of the noise power produced by the antenna; it is not the actual physical temperature of the antenna material.

SYSTEM NOISE

The antenna noise temperature is one of two components contributing to the overall *system noise* performance of a GPS receiver, which is a figure of merit of the whole receiving system. The other component is the receiver's equivalent noise temperature, which is a combination of cable losses and the noise internally generated in the receiver.

Cable Loss. The antenna noise temperature, T_a, must be corrected for the contribution by the cable between the antenna and the receiver or antenna preamplifier input. The cable is a "lossy" device: A signal traveling through it is attenuated. But not only does a lossy component reduce the signal level, it also adds to the noise. We can show that if L is the total loss in the cable (power in divided by power out; $L>1$), then the total antenna temperature is given by

$$T_{ant} = \frac{T_a}{L} + \frac{L-1}{L} T_0 \qquad [4]$$

where T_0 is the cable's ambient temperature. Alternatively, we may write this as

$$T_{ant} = \alpha T_a + (1-\alpha) T_0 \qquad [5]$$

where α, the fractional attenuation (0–1), is just the inverse of L. This equation is of the same form as the equation of radiative transfer found in physics. In fact, the physics of emission and absorption of electromagnetic radiation by a cloud of matter is similar to the emission and absorption taking place in an antenna cable.

Receiver Temperature. A receiver's noise temperature, T_r, is that of a noise source at the input of an ideal noiseless receiver that would produce the same level of output noise as the actual receiver's internal noise.

Instead of specifying the receiver's noise temperature, we can use the *noise factor, F,* where

$$F = \frac{N_{out}}{GkT_0B} \qquad [6]$$

and where N_{out} is the output noise power of the receiver, G its gain, and B its effective bandwidth. If a noise source connected to the receiver input has a noise temperature of T_0, the output's noise power is given by

$$N_{out} = GkT_0B + GkT_rB = Gk(T_0 + T_r)B. \qquad [7]$$

So that

$$F = 1 + \frac{T_r}{T_0}. \qquad [8]$$

Typically, T_0 is taken to be the standard reference temperature of 290 K. If T_r is also 290 K, for example, then $F = 2$. We can conveniently express the noise factor in decibels (dB), allowing us to correctly refer to it as a *noise figure*. For our example, the noise figure is 3.01 dB. Note that the term "noise figure" is often used arbitrarily for both the noise factor and its logarithm.

Radio receivers, including GPS receivers, are typically constructed as a number of stages, each with its own gain, contributing its own noise, and interconnected by cables or other circuitry. A GPS receiver might have, for example, a preamplifier in a separate antenna unit's base, connected to the receiver by a coaxial cable. Within the receiver proper, further stages of amplification exist. The total receiver temperature referenced to the input of the first amplifier stage is given by

$$T_r = \frac{T_0}{L_1}[L_1 - 1 + L_1[F_1 - 1 + G_1^{-1}[L_2 - 1 + L_2[F_2 - 1 + G_2^{-1}[L_3 - 1 + ...]]]]]. \quad [9]$$

We can refer the receiver noise to the antenna terminals, if we choose, by multiplying by L_l. Note that the loss and noise factor of the first stage, say the preamplifier in the antenna's base, dominates the receiver noise temperature. Any losses after this first amplification stage are reduced proportional to this stage's gain. If this gain is high enough, then we can write to a good approximation

$$T_{sys} = \frac{T_a}{L_1} + \frac{L_1 - 1}{L_1}T_0 + (F_1 - 1)T_0 . \quad [10]$$

Assuming a value of T_a = 130 K, L_l = 1.1, and a noise figure of the preamplifier of 1.5 dB (F_l = 1.41), we get a value for T_{sys} of a GPS receiving system of 118 + 26 + 119 = 263 K. The corresponding noise power density is 3.63 x 10^{-21} watts per hertz. Or, in logarithmic measure, –204 dBW-Hz.

CARRIER-TO-NOISE DENSITY RATIO

We now have a measure of the noise level that can contaminate a GPS observation, and we can compare it with the power of a GPS signal. In the absence of any GPS signal, the receiver and its associated antenna and preamplifier will detect a certain noise power, *N*. We can use the ratio of the power of a received signal, *S*, and the noise power, *N*, measured at the same time and place in a circuit as a measure of signal strength. Obviously, the larger the *S/N* value, the stronger the signal.

We usually make signal-to-noise measurements on signals at baseband (the band occupied by a signal after demodulation). At radio and intermediate frequencies (RF and IF), we commonly describe the signal level with respect to the noise level using the *carrier-to-noise-power-density ratio*, C/N_0. This is the ratio of the power level of the signal carrier to the noise power in a 1-Hz bandwidth. It is a key parameter in analyzing GPS receiver performance and directly affects the precision of the receiver's pseudorange and carrier-phase observations.

The expected minimum received C/A-code signal level is –160 dB. This is the signal level referenced to a 0-dB gain, isotropic, circularly polarized antenna. As mentioned earlier, an actual GPS receiver omnidirectional antenna may have a few decibels of gain near the zenith and negative gain at very low elevation angles with respect to a 0-dBic antenna. Also, 1 or 2 dB of cable and circuit losses will occur. So we may take –160 dBW as a strawman minimum carrier power level.

To determine the noise power density, we use the system temperature as described earlier.

Using the value of –160 dBW for the received C/A-code carrier power and ignoring signal gains and losses in the antenna, cable, and receiver, and using the value of -204 dBW-Hz for the noise density, we have a value for the carrier-to-noise-density ratio of about 44 dBW-Hz. Actually, C/N_0 values experienced in practice will vary a bit from this value. This variation will depend on the satellite transmitter's actual power output; changes in the space loss (caused by the spreading of the signal in space) with varying distance between satellite and receiver; differences in satellite and receiver antenna gain with elevation angle and azimuth of arriving signals; signal losses in the preamplifier, antenna cable, and receiver; and a small increase in effective noise caused by the signals of the other satellites in view.

All GPS satellites launched thus far have transmitted at levels where the received power has exceeded the minimum specified levels by as much as 5 or 6 dB. Nominal C/N_0 values are, therefore, usually above 45 dB-Hz, and most modern high-performance GPS receivers typically experience values of 50 dB-Hz or so.

Note that even a very strong C/A-code signal with a level of –150 dBW at the antenna terminals is buried in the ambient noise which, in the approximately 2-MHz C/A-code bandwidth, has a power of –141 dBW, some 9 dB stronger than the signal. Of course, the signal is raised out of the noise through the receiver's code-correlation process. A GPS receiver's *process* or *spreading gain* is theoretically the ratio of the transmitted signal's bandwidth to the navigation message data rate. For the C/A-code signal, this works out to be about 43 dB. So, after despreading, the *S/N* of the navigation message bits for a very strong signal would be about 34 dB. Actually, depending on a receiver's particular approach to processing the GPS signals, the processing gain could be a few decibels less than the theoretical value.

The C/N_0 value determines, in part, how well the receiver's tracking loops can track the signals and hence how precisely the receiver obtains pseudorange and carrier phase observations. In the following discussion, we will consider only the effect of noise on code and carrier-tracking loops in a standard code-correlating receiver.

CODE-TRACKING LOOP

The code-tracking loop — or delay lock loop, DLL — jitter for an early/late one-chip-spacing correlator is given by

$$\sigma_{DLL} = \sqrt{\frac{\alpha B_L}{c/n_0}\left[1+\frac{2}{T\,c/n_0}\right]}\lambda_c \qquad [11]$$

where α is the dimensionless DLL discriminator correlator factor (1 for a time-shared tau-dither early/late correlator, 0.5 for dedicated early and late correlators); B_L is the equivalent code loop noise bandwidth (Hz); c/n_0 is the carrier-to-noise density expressed as a ratio ($=10^{(C/N_0)/10}$ for C/N_0 expressed in dB-Hz); T is the predetection integration time (in seconds; T is the inverse of the predetection bandwidth or, in older receivers, the postcorrelator IF bandwidth); and λ_c is the wavelength of the PRN code (29.305 meters for the P-code; 293.05 meters for the C/A-code). The second term in the brackets in equation [11] represents the so-called squaring loss.

Typical B_L values for modern receivers range from less than 1 Hz to several Hz. If the code loop operates independently of the carrier-tracking loop, then the code loop bandwidth needs to be wide enough to accommodate receiver dynamics. However, if an estimate of the dynamics from the carrier-tracking loop aids the code loop, then the code loop can maintain lock without a wide bandwidth. Because the code loop bandwidth only needs to be wide enough to track the ionospheric divergence between pseudorange and carrier phase, it is not uncommon for carrier-aided receivers to have a code loop bandwidth on the order of 0.1 Hz.

Note that B_L in equation [11] is not necessarily the code loop bandwidth. If the receiver or external software performs post-measurement smoothing (or filtering) of the pseudoranges using the much lower noise carrier-phase observations, then

$$B_L = \frac{1}{2T_s} \qquad [12]$$

where T_s is the smoothing interval.

The predetection integration time, T, is typically 0.02 seconds (the navigation message data bit length). Increasing T reduces the squaring loss, which can be advantageous in weak-signal situations.

For moderate to strong signals ($C/N_0 \gtrsim 35$ dB-Hz), equation [11] is well approximated by

$$\sigma_{DLL} \approx \sqrt{\frac{\alpha B_L}{c/n_0}}\lambda_c\ . \qquad [13]$$

Using this approximation with α = 0.5, C/N_0 = 45 dB-Hz, and B_L = 0.8 Hz, σ_{DLL} for the C/A-code is 1.04 meters. The *S/N* in the code-tracking loop approximately equals c/n_0 divided by B_L. For our example, this works out to be about 46 dB.

The most recently developed high-performance GPS receivers use narrow correlators in which the spacing between the early and late versions of the receiver-generated reference code is less than one chip. For such receivers, we can rewrite equation [13] for signals of nominal strength as

$$\sigma_{DLL} \approx \sqrt{\frac{\alpha B_L d}{c/n_0}}\lambda_c \qquad [14]$$

where *d* is the correlator spacing in chips. For a spacing of 0.1 chips, and with the same values for the other parameters as used for the evaluation of the one chip correlator, σ_{DLL} for the C/A-code is 33 centimeters. With postmeasurement smoothing, a receiver can further reduce this jitter.

CARRIER-TRACKING LOOP

The analysis of the jitter in the carrier-tracking loop of a GPS receiver proceeds in a similar manner as that for the code-tracking loop. In fact, the expression for the jitter in a Costas-type phase lock loop has the same form as that for the code-tracking loop:

$$\sigma_{PLL} = \sqrt{\frac{B_P}{c/n_0}\left[1+\frac{1}{2T\,c/n_0}\right]}\frac{\lambda}{2\pi} \qquad [15]$$

where B_P is the carrier loop noise bandwidth (Hz), λ is the wavelength of the carrier, and the other symbols are the same as before. B_P must be wide enough for the tracking loop to follow the receiver dynamics. For most geodetic applications, for example, the receiver is stationary, so we can use bandwidths of 2 Hz or less. However, a tracking loop with such a narrow bandwidth might have problems following rapid variations in phase caused by the ionosphere. Some receivers adjust the loop bandwidth dynamically or allow the user to set the bandwidth manually.

For signals of nominal strength, equation [15] is well approximated by

$$\sigma_{PLL} = \sqrt{\frac{B_P}{c/n_0}}\frac{\lambda}{2\pi} \qquad [16]$$

Using this approximation with C/N_0 = 45 dB-Hz and B_P = 2 Hz, σ_{PLL} for the L1 carrier phase is 0.2 millimeter. We can approximate the *S/N* in the carrier-tracking loop in the same manner as for the code-tracking loop. For our example, this equals about 42 dB.

CONCLUSION

Noise is something we usually try to get away from in our everyday lives. And as we have seen in this brief article, noise is something we should also try to avoid in GPS receiving systems. The level of GPS receiving equipment system noise determines, in part, how precisely pseudoranges and carrier phases can be measured. Although we can't control much of the antenna noise caused by natural factors, we can minimize the overall system noise by ensuring that the receiver noise — principally that of the antenna preamplifier — is acceptably small.

We should also be aware that if we perform a zero-baseline test to assess the noise level of a GPS receiver's measurements, simply splitting the signal from the antenna preamplifier and feeding it directly into the antenna inputs of a pair of receivers will not provide an accurate indication of receiver performance level. The preamplifier, sky, and ground noises dominate the receiver system noise budget, and a conventional zero-baseline arrangement simply differs these noise sources away, resulting in an overly optimistic assessment of receiver performance. ■

Further Reading

For general discussions about antenna and receiver noise, see

■ *Satellite Communications,* 2d edition, by D. Roddy, published by McGraw-Hill, New York, New York, 1995.

■ *Radio Astronomy,* 2d edition, by J.D. Kraus, published by Cygnus-Quasar Books, Powell, Ohio, 1986.

For discussions about GPS antenna and receiver noise and its effect on the GPS observables, see

■ "GPS Signal Structure and Theoretical Performance," by J.J. Spilker, Jr., and "GPS Receivers," by A.J. Van Dierendonck in *Global Positioning System: Theory and Applications,* Vol. I, edited by B.W. Parkinson and J.J. Spilker, Jr., and published as Volume 163 of *Progress in Astronautics and Aeronautics,* by the American Institute of Aeronautics and Astronautics, Inc., Washington, D.C., 1996.

■ "Satellite Signal Acquisition and Tracking," by P. Ward, in *Understanding GPS: Principles and Applications,* edited by E.D. Kaplan and published by Artech House, Inc., Norwood, Massachusetts, 1996.

For a discussion of an appropriate method for assessing GPS receiver noise, see

■ "Measuring GPS Receiver Performance: A New Approach," by S. Gourevitch, in *GPS World,* Vol. 7, No. 10, October 1996, pp. 56–62.